

APPENDIX E
SAMPLE CALCULATIONS

SECTION I
FRANCIS TYPE TURBINES AND PUMP-TURBINES

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1. DESIGN REQUIREMENTS:

- a. Powerplant capacity - 150,000 KW
- b. Installation: 2 pump-turbines and one conventional turbine.
- c. Pumping requirements: 3,600 cfs each at a dynamic head of 153 feet. The heads vary between 137.5 and 160 feet. The minimum tailwater level for pumping is Elev. 540 ft. m.s.l.
- d. Generating requirements: The 3 units must have an aggregate dependable capacity of 150,000 KW. The 3 units must also be capable of producing 170,500 KW at a net head of 137.4 feet. The net heads vary between 132.5 and 151.9 feet. The average (rated) net head is 144.8 feet.

2. SELECTION OF PUMP-TURBINES.

a. Reference Figure 2, Appendix C to note that the recommended specific speed, N_{sp} for the 153 foot rated pumping head has a value of about 4,000.

b. Referring to the model curves in Appendix D, select the design shown on Figure PT3 as the best choice for this specific speed.

c. At maximum efficiency, note the following: $E_1 = 87.4$ percent, $Q_1 = 1.57$ cfs and $\phi_{TH} = 1.15$.

d. Calculate D_{TH} :

$$3,600 = 1.57 \left(\frac{D_{TH}}{12} \right)^2 (153)^{1/2}$$

$$D_{TH} = 163 \text{ inches}$$

e. Calculate speed, N:

$$N = \frac{1838 (1.15) (153)^{1/2}}{163} = 160 \text{ rpm}$$

Round to 163.6 rpm

f. The calculation of the runner throat diameter and associated synchronous speed generally requires an iterative solution. Further

iterations are required until the selected value for ϕ_{TH} and associated value of Q_1 produce a value for D_{TH} which substituted in the speed equation, step e, yields a synchronous speed. The following approximation is used to calculate the next trial value of ϕ_{TH} :

$$\phi_{TH} = 1.15 \frac{163.6}{160} = 1.176$$

The necessary iterations for this case are as follows:

Step	ϕ_{TH}	Q_1	D_{TH}	N
1	1.150	1.57	163.4	160.0
2	1.176	1.65	159.4	167.8
3	1.160	1.59	162.4	162.4
4	1.165	1.61	161.3	164.2
5	1.162	1.60	161.8	163.2

The accuracy in reading the model test data does not allow a closer determination of D_{TH} or N from step 5. Therefore, the solution indicates $D_{TH} = 162$ inches for $N = 163.6$ rpm.

g. At this point the user should make a cursory examination of the pumping efficiencies for other heads with a view to, perhaps, changing the speed to alter the head-efficiency characteristic. In this example, the following relationships are noted:

$$\phi_{TH} = \frac{163.6 (162)}{1838 (H)^{1/2}} = \frac{14.42}{H^{1/2}}$$

H	ϕ_{TH}	E_1
160.0	1.140	87.4
153.0	1.166	87.3
137.5	1.230	86.3

This relationship is satisfactory and the balance of the example is completed on the basis of $D_{TH} = 162$ inches and $N = 163.6$ rpm.

h. Calculate efficiency step-up.

$$E_2 = 100 - (100 - E_1) \left(\frac{D_m}{D_p} \right)^{0.2}$$

Where, max. E_1 = 88.5 percent (generating); D_m = 12 inches;
 D_p = 162 inches.

$$E_2 = 100 - (100 - 88.5) \left(\frac{12}{162} \right)^{0.2} = 93.2 \text{ percent}$$

$$\text{step-up} = (2/3) (93.2 - 88.5) = 3.1 \text{ percent}$$

i. The expected pumping discharge is calculated to include the effect of the higher prototype expected efficiency as follows:

$$Q_{2C} = Q_1 \left(\frac{162}{12} \right)^2 (H)^{1/2} \frac{E_2}{E_1}$$

j. The required pumping horsepower is calculated as follows:

$$HP = \frac{Q_{2C} H_w}{550 E_2}$$

k. The required setting of the runner is controlled by the maximum head-minimum tailwater condition. For maximum head, $\phi_{TH} = 1.14$ and from Figure PT3, $C = 0.295$.

$$\sigma_C = \frac{H_b - H_v - H_s - \text{safety}}{H}$$

Refer to Figure 6, Appendix C: For tailwater Elev. 540, H_b = 33.3 feet and a water temperature of 70° F., H_v = 0.8 feet.

$$\text{safety margin} = 0.2 D_i + 0.4 H^{1/2}$$

Refer to Table 3, Appendix C, noting that $D_i = 1.154$

$$\text{therefore, } D_i = 1.154 \frac{162}{12} = 15.6 \text{ feet}$$

$$\text{subs: safety margin} = 0.2 (15.6) + 0.4 (160)^{1/2} = 8.2 \text{ feet}$$

The required submergence is calculated as follows:

$$0.295 = \frac{33.3 - 0.8 - H_s - 8.2}{160}$$

$$H_s = -22.9 \text{ feet}$$

The distance, a , between the bottom of the runner and the distributor centerline is calculated using the ratio, d , from Table 3, Appendix C, as follows:

$$d = 0.385$$

$$a = 0.385 (162/12) = 5.2 \text{ feet}$$

The elevation of the distributor centerline is calculated as follows:

$$\text{Elev.} = \text{tailwater Elev.} + H_s + a$$

$$\text{Elev.} = 540 + (-22.9) + 5.2 = 522.3 \text{ ft. m.s.l.}$$

1. The expected pumping performance is as follows:

Head	137.5	145.0	153.0	160.0
Q_{TH}	1.230	1.198	1.166	1.140
Q_1	1.82	1.72	1.62	1.53
E_1	86.3	86.8	87.3	87.4
Q_{2C}	4,029	3,909	3,777	3,652
E_2	89.4	89.9	90.4	90.5
HP	70,200	71,420	72,410	73,140

3. GENERATING CYCLE.

a. The prototype expected performance is calculated as follows:

$$\phi_{TH} = \frac{162 (163.6)}{1838 (H)^{1/2}} = \frac{14.42}{(H)^{1/2}}$$

$$HP_2 = HP_1 \left(\frac{162}{12} \right)^2 (H)^{3/2} = 182.25 (HP_1) (H)^{3/2}$$

$$Q_2 = \frac{550 HP_2}{62.3 (H) E_2} = 8.828 \frac{HP_2}{H E_2}$$

$$E_2 = E_1 + 3.1 \text{ percent}$$

Head	ϕ_{TH}	Percent gate	HP_1	E_1	HP_2	Q_2	E_2
132.5	1.253	100	0.204	83.3	56,710	4,375	86.4
		90	0.198	84.6	55,040	4,180	87.7
		80	0.185	83.9	51,420	3,940	87.0
		70	0.164	80.9	45,590	3,615	84.0
		60	0.140	76.0	38,920	3,280	79.1
137.4	1.230	100	0.203	83.5	59,590	4,420	86.6
		90	0.198	85.0	58,120	4,240	88.1
		80	0.187	84.7	54,890	4,015	87.8
		70	0.166	82.1	48,730	3,675	85.2
		60	0.143	78.0	41,970	3,325	81.1
145	1.198	100	0.202	83.6	64,280	4,515	86.7
		90	0.197	85.2	62,690	4,320	88.3

Head	ϕ_{TH}	Percent gate	HP ₁	E ₁	HP ₂	Q ₂	E ₂
151.9	1.170	80	0.188	85.9	59,820	4,090	89.0
		70	0.169	83.8	53,780	3,765	86.9
		60	0.147	80.0	46,780	3,425	83.1
		50	0.119	74.0	37,870	2,990	77.1
		100	0.200	83.5	68,240	4,580	86.6
		90	0.197	85.2	67,220	4,425	88.3
		80	0.189	86.5	64,490	4,180	89.6
		70	0.171	85.0	58,340	3,850	88.1
		60	0.150	81.8	51,180	3,505	84.9
		50	0.123	76.8	41,970	3,055	79.9

b. The maximum runaway speed is calculated as follows:

Refer Figure PT3 to note that $\phi_{max} = 2.09$

$$N_{max} = \frac{1838 (2.09) (151.9)^{1/2}}{162} = 292 \text{ rpm}$$

c. The guaranteed capacities at the 132.5 foot and 137.4 foot net head conditions are calculated at 98 percent of the 100 percent gate capacities indicated in above tabulation. The guaranteed capacities for the conventional unit at these two heads are as follows:

$$\text{KW output} = 0.98 (0.746) E_g \text{ HP}_2$$

$$\text{KW output} = 0.98 (0.746) (0.97) \text{ HP}_2 = 0.709 \text{ HP}_2$$

Head - feet	132.5	137.4
Plant output - KW	150,000	170,500
Pump-turbines - KW	80,400	84,500

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Conventional - KW	69,600	86,000
Conventional - HP	96,180	118,850
Expected - HP	98,140	121,280

4. CONVENTIONAL FRANCIS TURBINE.

a. The relationship for N_s vs. Head shown on Figure 1, Appendix C insures designs with moderate speeds and relatively shallow submergences. In a mixed installation with pump-turbines and conventional turbines, the inherent deeper submergences required of the former generally dictates a variation of this conservative approach. This is necessary to provide a more balanced equipment layout and avoid exaggerated levels for the generator-motors and generators. For this reason the "K" value used in Figure 1 is increased to, say, a value of 800. The corresponding N_s for $H = 144.8$ feet is 66.5.

b. Referring to the model curves in Appendix D, it may be noted that the designs shown on Figures F6 and F7 are within the range of this specific speed. A comparison of these designs indicates that the former has higher unit power with attendant higher critical sigmas, whereas the latter has higher overall efficiencies with lower critical sigmas and reduced unit power. The former design, Figure F6, is selected for the following reasons. The higher unit power will result in a smaller runner throat diameter with consequent smaller physical dimensions of the turbine to more nearly approach the physical dimensions of the pump-turbines. The higher critical sigmas require deeper submergences, however, this is not inappropriate in view of the deep submergence of the pump-turbines.

c. The method for sizing this unit differs from the conventional approach for Francis turbines. In this instance, the output required at the 137.4 feet critical net head dictates the size. This output is associated with the full gate capacity at a value of ϕ_{TH} slightly higher than the best ϕ_{TH} to be associated with the average head of 144.8 feet. For the latter condition a first value of $\phi_{TH} = 0.86$ is chosen. The corresponding value for the 137.4 foot head condition is calculated as follows:

$$\phi_{TH} = 0.86 \left(\frac{144.8}{137.4} \right)^{1/2} = 0.883$$

From Figure F6 for $\phi_{TH} = 0.883$, the 100 percent gate HP = 0.29. This is associated with the required expected output of 121,280 HP to

calculate D_{TH} as follows:

$$121,280 = 0.29 \left(\frac{D_{TH}}{12} \right)^2 (137.4)^{3/2}$$

$$D_{TH} = 193.4 \text{ inches}$$

d. Calculate the speed, as follows:

$$N = \frac{1838 (0.833) (137.4)^{1/2}}{193.4} = 98.4$$

Round to nearest synchronous speed = 100 rpm

This speed and the D_{TH} calculated above are first values of an iterative solution similar to that described in 2.f. of this example. The necessary iterative steps are as follows:

Step	ϕ_{TH}	HP ₁	D_{TH}	N
1	0.883	0.29	193.4	98.4
2	0.897	0.29	193.4	100

Round D_{TH} to 193.5 inches

e. The expected prototype output is calculated as follows:

$$HP_2 = HP_1 \left(\frac{193.5}{12} \right)^2 (H)^{3/2} = 260.02 (HP_1) (H)^{3/2}$$

f. The efficiency step-up is calculated, using the procedure established in 2.h. of this example, as follows:

$$E_2 = 100 - (100 - 90) \left(\frac{12}{193.5} \right)^{0.2}$$

$$E_2 = 94.3 \text{ percent}$$

$$\text{step-up} = (2/3) (94.3 - 90) = 2.9 \text{ percent}$$

g. The expected discharge is calculated as follows:

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$$Q_2 = \frac{550 \text{ HP}_2}{62.3 (H) E_2} = 8.828 \frac{\text{HP}_2}{(H) E_2}$$

h. The guaranteed capacity required at the 137.4 feet critical head is 118,850 HP. The generator output is 86,000 KW. The generator nameplate rating is 86,000 KW at 0.95 p.f. or 90,526 KVA. The turbine is designed to mechanically withstand operation at the generator nameplate rating at 1.0 p.f. or 125,100 HP. The turbine setting is predicated on the availability of 118,850 HP at the critical and higher heads. Although the critical head conditions will generally dictate the setting, it is recommended that other conditions be checked to assure that the critical sigma characteristics of proposed design or unusual tailwater conditions do not alter this normal circumstance.

i. The procedure described in 2.k. of this example is used to establish the turbine setting. The dimensionless ratios for calculating the dimensions D_1 and a are obtained from Table 1, Appendix C. The results of pertinent calculations are tabulated as follows:

Net head, feet	137.4	144.8	151.9
T.W. elev., ft.m.s.l.	550.8	548.5	541.6
HP	118,850	118,850	118,850
ϕ_{TH}	0.898	0.875	0.854
HP_1	0.284	0.262	0.244
σ_C	0.245	0.1830	0.160
H_b , feet	33.3	33.3	33.3
H_v , feet	0.8	0.8	0.8
D_1 , feet	14.2	14.2	14.2
Safety margin, feet	7.5	7.7	7.8
H_s , feet	-8.7	-1.7	+0.4
a , feet	6.9	6.9	6.9

Dist. elev, ft. m.s.l. 549.0 553.7 548.9

It is to be noted that the conditions at the critical and maximum heads dictate about the same setting.

j. The maximum runaway speed is calculated as follows:

From Figure F6, $\phi_{\max} = 1.671$

$$N_{\max} = \frac{1838 (1.671) (151.9)^{1/2}}{193.5} = 195.6 \text{ rpm}$$

k. The expected prototype performance is tabulated below:

Head	ϕ_{TH}	HP ₁	E ₁	HP ₂	Q ₂	E ₂
132.5	0.915	0.123	75	48,780	77.9	4,170
		0.148	80	58,690	82.9	4,715
		0.185	84	73,370	86.9	5,625
		0.217	87	86,060	89.9	6,375
		0.241	89	95,570	91.9	6,930
		0.258	89	102,320	91.9	7,415
		0.273	87	108,260	89.9	8,025
		0.285	84	113,020	86.9	8,665
		0.290	82.5	115,010	85.4	8,970
137.4	0.898	0.122	75	51,090	77.9	4,215
		0.148	80	61,980	82.9	4,805
		0.211	87	88,360	89.9	6,315

Head	ϕ_{TH}	HP ₁	E ₁	HP ₂	Q ₂	E ₂
144.8	0.875	0.235	89	98,410	91.9	6,880
		0.259	89	108,460	91.9	7,585
		0.274	87	114,740	89.9	8,200
		0.285	84	119,350	86.9	8,825
		0.120	75	54,370	77.9	4,255
		0.146	80	66,150	82.9	4,865
		0.174	84	78,830	86.9	5,530
		0.203	87	91,970	89.9	6,235
		0.227	89	102,840	91.9	6,825
		0.260	89	117,790	91.9	7,815
151.9	0.854	0.273	87	123,680	89.9	8,390
		0.119	75	57,930	77.9	4,320
		0.144	80	70,100	82.9	4,915
		0.170	84	82,750	86.9	5,535
		0.197	87	95,900	89.9	6,200
		0.219	89	106,610	91.9	6,740
		0.239	90	116,340	92.9	7,280
		0.246	90	119,750	92.9	7,490

5. PROTOTYPE DIMENSIONS. The prototype dimensions of the pump-turbines can be calculated from the dimensionless ratios shown in Table 3, Appendix C. Similar dimensions for the Francis turbine can be calculated from the ratios shown in Table 1, Appendix C.

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SECTION II
FIXED BLADE PROPELLER TURBINE

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1. DESIGN REQUIREMENTS.

a. Powerplant capacity: 65,000 KW with 2 units.

b. Generator requirements:

(1) Nameplate rating: 36,111 KVA, 0.9 pf., 32,500 KW, 13.8 KV and 60 Hz.

(2) Must be designed for continuous operation at rated KVA, voltage, p.f. and frequency.

c. Turbine requirements:

(1) Net heads: 78 foot rated, 60 foot minimum and 100 foot maximum.

(2) Require 30,000 HP guaranteed output at 60 foot net head.

(3) Turbine output is limited to 49,900 HP (rated KVA at 1.0 p.f.).

2. TURBINE SELECTION.

a. To utilize the capability of a generator mated with a fixed blade propeller turbine, the turbine at or near best efficiency at rated head should have an output of 95 percent of the horsepower equivalent of the generator rating:

$$HP = \frac{0.95 (32,500)}{0.746 (0.97)} = 42,600$$

b. Reference Figure 3, Appendix C to note that the rated head condition and the wide head range for this unit dictates a 6 blade runner. The recommended specific speed at the 78 foot rated head is calculated as follows:

$$N_s = \frac{1,000}{(78)^{1/2}} = 113.2$$

c. The speed is calculated as follows:

$$N = \frac{113.2 (78)^{5/4}}{(42,600)^{1/2}} = 127.1$$

Round to nearest synchronous speed = 128.6 rpm

Corrected $N_s = 114.5$

d. For preliminary studies requiring only an approximate speed and runner throat diameter, the following empirical formula for ϕ_{TH} may be used to calculate the diameter:

$$\phi_{TH} = 0.089 (114.5)^{0.58} = 1.391$$

$$D_{TH} = \frac{1838 (1.391) (78)^{1/2}}{128.6} = 175.6 \text{ inches}$$

e. The appropriate model test curves for these conditions are shown on Figure FB3. A curve of best efficiency is constructed from the following data taken from the efficiency contours:

ϕ_{TH}	1.230	1.290	1.350	1.420	1.515
HP ₁	0.244	0.256	0.264	0.272	0.290

The location of the design point along this curve is determined by iteration. This is accomplished by substituting associated values of HP₁ and ϕ_{TH} in the following formula for specific speed:

$$N_s = 153.17 (\phi_{TH}) (HP_1)^{1/2} = 114.5$$

The approximate value $\phi_{TH} = 1.391$ from (d) above is used in the first step of the iterative process as follows:

ϕ_{TH}	1.391	1.440	1.430
HP ₁	0.2685	0.2745	0.2730
N_s	110.4	115.6	114.5

The design point is located at $\phi_{TH} = 1.430$ and $HP_1 = 0.2730$. The runner throat diameter for this preliminary selection is calculated as follows:

$$42,600 = 0.273 \left(\frac{D_{TH}}{12} \right)^2 (78)^{3/2}$$

$$D_{TH} = 180.6 \text{ inches}$$

f. It may be noted from inspection of Figure FB3 that the design point calculated above is located to the right of bet The model efficiency at this point is $E_1 = 87.9$ percent, which is less than the 88.4 percent peak efficiency. The peak efficiency at the rated conditions can be improved by selecting the next lower synchronous speed, 120 rpm, and repeating the iterative solution for the new design point. The calculations and tabulation of the iterative steps are as follows:

$$N_s = \frac{(42,000)^{1/2}}{(78)^{5/4}} (120) = 106.8$$

$$\text{Use first trial } \phi_{TH} = 1.43 \left(\frac{120}{128.6} \right) = 1.334$$

ϕ_{TH}	1.334	1.362	1.355
HP_1	0.2620	0.2650	0.2645
N_s	104.6	107.4	106.7

Calculate the runner throat diameter:

$$42,600 = 0.2645 \left(\frac{D_{TH}}{12} \right)^2 (78)^{3/2}$$

$$D_{TH} = 183.5 \text{ inches}$$

g. The second selection matches the peak efficiency of this design to the rated conditions. This is accomplished by selecting a larger, lower speed unit. At this point in the selection process, the user must

evaluate the increased capital costs of the larger unit against the benefits of the higher efficiency. The costs should include the effects on the powerhouse structure, excavation taking into account any change in the turbine setting, generator cost, . . . etc. The latter selection is arbitrarily used in the remainder of this example.

h. The model test curves must be checked to assure that the 30,000 HP guaranteed output at 60 foot minimum net head can be developed with the proposed design. The necessary calculations in this determination are as follows:

$$\phi_{TH} = \frac{120 (183.5)}{1838 (H)^{1/2}} = \frac{11.98}{(H)^{1/2}} = 1.547$$

$$30,000 = HP_1 \left(\frac{183.5}{12} \right)^2 (60)^{3/2}$$

$$HP_1 = 0.2760$$

Referring to Figure FB3 at $\phi_{TH} = 1.547$, note that the full gate (100 percent) output is $HP_1 = 0.3070$.

$$\text{percent margin} = \frac{0.307}{0.276} (100) = 111.2 \text{ percent}$$

The design meets the requirement that the expected full gate output is at least 2 percent greater than the guaranteed output.

i. The prototype expected performance is calculated as follows:

$$\phi_{TH} = \frac{11.98}{(H)^{1/2}}$$

$$HP_2 = HP_1 \left(\frac{183.5}{12} \right)^2 (H_2)^{3/2} = 233.84 (HP_1) (H_2)^{3/2}$$

$$E_2 = 100 - (100 - 88.4) \left(\frac{12}{183.5} \right)^{0.2} = 93.3 \text{ percent}$$

$$\text{step-up} = (2/3) (93.3 - 88.4) = 3.3 \text{ percent}$$

$$E_C = E_1 + 3.3 \text{ percent}$$

$$Q_2 = \frac{550 \text{ HP}_2}{62.3 \text{ E}_c \text{ H}_2} = 8.828 \frac{\text{HP}_2}{\text{E}_c \text{ H}_2}$$

H ₂	Ø _{TH}	HP ₁	E ₁	HP ₂	E _c	Q ₂
60	1.547	0.184	70	20,000	73.3	4,015
		0.208	75	22,610	78.3	4,015
		0.234	80	25,430	83.3	4,490
		0.247	82	26,840	85.3	4,630
		0.264	84	28,690	87.3	4,835
		0.272	85	29,560	88.3	4,925
		0.282	86	30,650	89.3	5,050
		0.304	86	33,040	89.3	5,445
		0.307	85	33,360	88.3	5,560
78	1.356	0.159	70	25,610	73.3	3,955
		0.181	75	29,160	78.3	4,215
		0.204	80	32,860	83.3	4,465
		0.214	82	34,470	85.3	4,575
		0.226	84	36,410	87.3	4,720
		0.231	85	37,210	88.3	4,770
		0.238	86	38,340	89.3	4,860
		0.244	87	39,310	90.3	4,925
		0.254	88	40,920	91.3	5,070

H_2	ϕ_{TH}	HP_I	E_1	HP_2	E_C	Q_2
		0.264	88.4	42,530	91.7	5,250
		0.271	88	43,650	91.3	5,410
		0.278	87	44,780	90.3	5,615
		0.282	86	45,430	89.3	5,755
		0.285	85	45,910	88.3	5,885
		0.287	84.4	46,230	87.7	5,965
100	1.198	0.135	70	31,570	73.3	3,800
		0.165	75	38,580	78.3	4,350
		0.191	80	44,660	83.3	4,735
		0.202	82	47,240	85.3	4,890
		0.213	84	49,810	87.3	5,035
		0.219	85	51,210	88.3	5,120

j. The setting of the turbine depends upon the output requirements and the related head-tailwater conditions. The setting is generally predicated on the tailwater level with one unit operating. For this example it is assumed that it is desired to operate the unit at generator rating and 0.9 p.f. under the rated and higher heads. The corresponding turbine output is 44,500 HP. Under normal circumstances the rated condition dictates the setting. However, it is good practice to check the other head conditions to assure that unusual sigma characteristics or head-tailwater relationships do not alter this normal circumstance. The output requirements at the lower heads are assumed to vary directly with the head between the 44,500 HP at 78 foot and the 30,000 HP guaranteed output at 60 foot head. The relationship between tailwater level and discharge is linear between the following sets of conditions:

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Tailwater Elev.- ft.m.s.l. 543.0 545.7

Discharge - cfs 5,000 11,000

k. The following steps are required to establish the turbine settings on the basis of the conditions set forth above:

$$\phi_{TH} = \frac{11.98}{(H)^{1/2}}$$

$$HP_1 = \frac{HP_2}{233.84 H^{3/2}}$$

From Figure FB3, pick off σ'_C and E_1

$$E_C = E_1 + 3.3 \text{ percent}$$

$$Q_2 = \frac{8.828 (HP_2)}{E_C (H_2)}$$

$$\text{T.W. Elev.} = \frac{Q_2}{2222} + 540.75$$

$$\sigma_C = \frac{H_b - H_v - H_s - \text{safety}}{H_2}$$

From Figure 6, Appendix C: $H_b = 33.3$ feet, $H_v = 0.8$ feet (70° F.)

$$\text{safety} = 0.2 D_{TH} + 0.7 (H_2)^{1/2}$$

$$\text{Distr. centerline elev.} = \text{T.W. elev.} + H_s + a$$

$$a = (d)D_{TH}$$

Refer to Table 4 and Figure 5, Appendix C to note $d = 0.365$

$$a = 0.365 (183.5/12) = 5.6 \text{ feet}$$

ϕ_{TH}	1.547	1.432	1.356	1.263	1.198
HP ₂	30,000	38,600	44,500	44,500	44,500
HP ₁	0.276	0.282	0.276	0.223	0.190
E ₁	85.4	87.4	87.3	85.1	80.0
c	0.375	0.365	0.385		
E _C	88.7	90.7	90.6	88.4	83.3
Q ₂	4,975	5,370	5,560	4,935	4,715
T.W. Elev.	543.0	543.2	543.3	543.0	542.9
Safety	8.5	8.9	9.2	9.7	10.1
H _S	1.5	-2.0	-6.8		
Distr. Elev.	550.1	546.8	542.1		

l. As generally expected, the rated conditions dictate the turbine setting. The HP₁ values shown in above tabulation at the higher heads are well below the range of sigma values shown on Figure FB3. This is due to the fact that HP₁ varies with the inverse of $H^{3/2}$. Since the tailwater levels do not vary substantially at the higher heads, the plant sigma with the distributor set at Elevation 542.1 varies only with the inverse of H and sufficient submergence is assured.

m. The cavitation limits for the higher heads can be established for the selected setting by deriving a relationship for c in terms of head then entering the critical sigma curves on Figure FB3 to estimate the corresponding value of HP₁. This procedure is as follows:

$$\phi_{TH} = \frac{1.98}{(H_2)^{1/2}}$$

$$\sigma_C = \frac{H_b - H_v - (\text{Distr. El.} - \text{T.W. El.} - a) - \text{safety}}{H_2}$$

By substituting known values and allowing a constant tailwater level at Elev. 543, this equation becomes:

$$c = \frac{35.9 - 0.7 (H_2)^{1/2}}{H_2}$$

A summary of the maximum output limits is as follows:

H_2	ϕ_{TH}	c	HP ₁	HP ₂
80	1.339	0.370	0.270	45,180
82	1.323	0.361	0.268	46,530
84	1.307	0.351	0.266	47,890
86	1.292	0.342	0.264	49,230
86.5	1.288	0.340	0.264	49,660

The limiting output of 49,500 HP can be developed at 86.5 feet and the higher heads without cavitation.

n. The prototype maximum runaway speed is estimated as follows:

$$N_{max} = \frac{1838 \phi_{max} (H)^{1/2}}{D_{TH}}$$

Refer to Figure FB3 to note that $\phi_{max} = 2.765$

$$N_{max} = \frac{1838 (2.765)(100)^{1/2}}{183.5} = 277 \text{ rpm}$$

o. As an exercise, the user may elect to analyze the merits of the first selection with $N = 128.6 \text{ rpm}$ and $D_{TH} = 180.6 \text{ inches}$. This will familiarize the user with the formulas and procedures required to develop the necessary data for a given design.

p. The prototype dimensions of the principal parts and water passages of the turbine can be calculated from the dimensionless ratios shown in Table 4 Appendix C.

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APPENDIX E

SECTION III

ADJUSTABLE BLADE PROPELLER TURBINE

<u>SECTION</u>	<u>SUBJECT</u>	<u>PAGE</u>
1	DESIGN REQUIREMENTS	E-29
2	TURBINE SELECTION	E-29

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1. DESIGN REQUIREMENTS.

- a. Powerplant capacity: 69,000 KW with 2 units
- b. Generator requirements:
 - (1) Nameplate rating: 36,320 KVA, 0.95 p.f., 34,500 KW, 13.8 KV and 60 hz.
 - (2) Must be designed for continuous operation at rated KVA at rated voltage, p.f. and frequency.
- c. Turbine requirements:
 - (1) Net heads: 70 foot rated, 53 foot minimum and 88 foot maximum.
 - (2) Require 33,200 HP guaranteed output at 53 foot net head.
 - (3) Turbine output is limited to 50,190 HP (rated KVA at 1.0 p.f.)
 - (4) The setting requires a concrete semi-spiral case.

2. TURBINE SELECTION.

- a. The turbine output required to match the generator rating is calculated as follows:

$$HP = \frac{34,500}{0.746 (0.97)} = 47,680$$

- b. Refer to Figure 3, Appendix C to note that the rated head condition and the wide head range for this unit dictates a 6 blade runner. The recommended specific speed at the 70 foot rated head is calculated as follows:

$$N_s = \frac{1,100}{(70)^{1/2}} = 131.5$$

- c. The speed is calculated as follows:

$$N = \frac{131.5 (70)^{5/4}}{(47,680)^{1/2}} = 121.9$$

Round to nearest synchronous speed = 120 rpm

$$\text{Corrected } N_s = 129.5$$

d. For preliminary studies requiring only an approximate speed and runner throat diameter, the following empirical formula for ϕ_{TH} may be used to calculate the diameter:

$$\phi_{TH} = 0.049 (129.5)^{0.695} = 1.440$$

$$D_{TH} = \frac{1838 (1.440) (70)^{1/2}}{120} = 184.5 \text{ inches}$$

e. The appropriate model test curves for these conditions are shown on Figure K3, Appendix D. The design point for rated conditions is to be located along the on-cam 32° blade angle curve. The location of the design point is determined by iteration. This is accomplished by substituting associated values of HP_1 and ϕ_{TH} in the following formula for specific speed:

$$N_s = 153.17 \phi_{TH} (HP_1)^{1/2}$$

The approximate value $\phi_{TH} = 1.440$ from d above is used in the first step of the iterative process as follows:

ϕ_{TH}	1.440	1.450	1.445
HP_1	0.342	0.343	0.342
N_s	129.0	130.1	129.4

The design point is located at $\phi_{TH} = 1.445$ and $HP_1 = 0.342$. The runner throat diameter for this preliminary selection is calculated as follows:

$$47,680 = 0.342 \left(\frac{D_{TH}}{12} \right)^2 (70)^{3/2}$$

$$D_{TH} = 185 \text{ inches}$$

f. The location of this design point with reference to the extremes in head conditions should be checked as follows:

$$\phi = \frac{120 (185)}{1838 (H_2)^{1/2}} = \frac{12.08}{(H_2)^{1/2}}$$

$$HP_1 = \frac{(HP_2)}{(185/12) (H)_2^{3/2}} = \frac{(HP_2)}{(237.67)^{3/2}}$$

(1) At the 53 foot minimum head a guaranteed output of 33,200 HP is required:

$$\phi_{TH} = \frac{12.08}{(53)^{1/2}} = 1.659$$

$$HP_1 = \frac{33,200}{237.67 (53)^{3/2}} = 0.362$$

Refer to Figure K3 at $\phi_{TH} = 1.659$ to note that the full gate (100 percent) $HP_1 = 0.405$

$$\text{percent margin} = \frac{0.405}{0.362} (100) = 111.9 \text{ percent}$$

The design meets the requirement that the expected full gate output is at least 2 percent greater than the guaranteed output.

(2) For the 88 foot maximum head, check the efficiencies for generator rated load:

$$\phi_{TH} = \frac{12.08}{(88)^{1/2}} = 1.288$$

$$\text{Rated } HP_1 = \frac{47,680}{237.67 (88)^{3/2}} = 0.243$$

From Figure K3, $E_1 = 88.1$ percent
This efficiency is considered satisfactory.

(3) From Figure K3 it is noted that the best $\phi_{TH} = 1.35$, which corresponds to a net head of 80.1 feet. At maximum efficiency, $E_1 = 89.6$ percent, the corresponding prototype expected output is 32,370 HP. This corresponds to about 68 percent of generator rated load.

g. It is recommended that alternate designs be investigated before making a final selection. In this instance the adjacent synchronous speeds, 112.5 and 128.6 rpm. should be investigated. The balance of this example, however, will proceed on the basis of $N = 120$ rpm and $D_{TH} = 185$ inches.

h. The prototype expected performance is calculated as follows:

$$\phi_{TH} = \frac{12.08}{(H)^{1/2}}$$

$$HP_2 = HP_1 (185/12)^2 = 237.67 HP_1 (H_2)^{3/2}$$

$$E_2 = 100 - (100 - 89.6) (12/185)^{0.2} = 94 \text{ percent}$$

$$\text{set-up} = (2/3) (94.0 - 89.6) = 2.9 \text{ percent}$$

$$E_C = E_1 + 2.9 \text{ percent}$$

$$Q_2 = \frac{550 HP_2}{62.3 (E_C) H_2} = 8.828 \frac{HP_2}{E_2 H_2}$$

H_2	ϕ_{TH}	HP_1	E_1	HP_2	E_C	Q_2
53	1.659	0.072	76	6,600	78.9	1,390
		0.080	78	7,340	80.9	1,505
		0.086	80	7,890	82.9	1,580
		0.098	82	8,990	84.9	1,760
		0.115	84	10,550	86.9	2,015
		0.151	86	13,850	88.9	2,590
		0.184	87	16,870	89.9	3,120

H ₂	Ø _{TH}	HP ₁	E ₁	HP ₂	E _C	Q ₂
70	1.444	0.276	87	25,310	89.9	4,680
		0.298	86	27,330	88.9	5,110
		0.327	84	29,990	86.9	5,735
		0.352	82	32,280	84.9	6,320
		0.372	80	34,110	82.9	6,840
		0.058	76	8,070	78.9	1,285
		0.063	78	8,770	80.9	1,385
		0.070	80	9,740	82.9	1,480
		0.078	82	10,860	84.9	1,610
		0.091	84	12,670	86.9	1,835
		0.110	86	15,310	88.9	2,165
		0.124	87	17,260	89.9	2,415
		0.145	88	20,180	90.9	2,795
		0.158	88.5	21,990	91.4	3,030
		0.173	89	24,080	91.9	3,300
		0.233	89	32,430	91.9	4,440
		0.249	88.5	34,660	91.4	4,770
		0.271	88	37,720	90.0	5,220
		0.302	87	42,040	89.9	5,885
		0.324	86	45,100	88.9	6,385
		0.355	84	49,410	86.9	7,155

H ₂	Ø _{TH}	HP ₁	E ₁	HP ₂	E _C	Q ₂
88	1.288	0.060	80	11,770	82.9	1,420
		0.069	82	13,540	84.9	1,595
		0.082	84	16,090	86.9	1,855
		0.101	86	19,820	88.9	2,230
		0.115	87	22,560	89.9	2,510
		0.134	88	26,290	90.9	2,895
		0.144	88.5	28,250	91.4	3,095
		0.158	89	31,000	91.9	3,375
		0.190	89.5	37,280	92.4	4,040
		0.218	89	42,770	91.9	4,660
		0.232	88.5	45,520	91.4	4,985
		0.245	88	48,070	90.9	5,290
		0.270	87	52,970	89.9	5,900

h. The setting of the turbine depends upon the output requirements and the related head-tailwater conditions. The setting is generally predicated on the tailwater level with one unit operating. For this example it is assumed that it is desired to operate the unit at the generator rating of 0.95 p.f. under the rated and higher heads. The corresponding turbine output is 47,680 HP. Under normal circumstances the rated condition dictates the setting. However, it is good practice to check the other head conditions to assure that unusual sigma characteristics or head-tailwater relationships do not alter this normal circumstance. The output requirements at the lower heads are assumed to vary directly with the head between the 47,680 HP at 70 foot and the 33,200 HP guaranteed output at 53 foot head. The relationship between tailwater level and discharge is linear between the following sets of conditions:

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Tailwater Elev.- ft. m.s.l.	500.0	543.5
Discharge- cfs	5,000	11,000

i. The following steps are required to establish the turbine settings on the basis of the conditions set forth above:

$$\phi_{TH} = \frac{12.08}{(H_2)^{1/2}}$$

$$HP_1 = \frac{HP_2}{237.67 (H_2)^{3/2}}$$

From Figure K3, pick off σ_C and E_1 for above ϕ_{TH} and HP_1 values.

$$E_C = E_1 + 2.9 \text{ percent}$$

$$Q_2 = 8.828 \frac{HP_2}{E_C H_2}$$

$$\text{T.W. Elev.} = \frac{Q_2}{1714} + 537.1$$

$$\sigma_C = \frac{H_b - H_v - H_s - \text{safety}}{H_2}$$

From Figure 6, Appendix C: $H_b = 33.3$ feet, $H_v = 0.8$ feet (70o F.)

$$\text{safety} = 0.2 D_{TH} + 0.7 (H_2)^{1/2}$$

Distributor centerline Elevation = T.W. Elevation + H_s + a

$$a = (d) D_{TH}$$

Refer Table 4 and Figure 5, Appendix C to note that $d = 0.368$

$$a = 0.368 (185/12) = 5.7 \text{ feet}$$

Refer to Figure S9 for values of critical runner sigma.

H_2	53	62	70	80	88
ϕ_{TH}	1.659	1.534	1.444	1.351	1.288
HP_2	33,200	40,870	47,680	47,680	47,680
HP_1	0.362	0.352	0.342	0.280	0.243
E_1	81.0	83.6	84.7	87.3	88.2
c	0.880	0.780	0.725	0.475	0.375
E_C	83.9	86.5	87.6	90.2	91.1
Q_2	6,590	6,730	6,860	5,830	5,250
T.W. Elev.	540.9	541.0	541.1	540.5	540.2
Safety	8.2	8.6	9.0	9.4	9.7
H_s	-22.3	-24.5	-27.2	-14.9	-10.2
Distr. Elev.	524.3	522.3	519.6	531.3	535.7

j. As generally expected, the rated conditions dictate the turbine setting. This is due to the fact that HP_1 varies with the inverse of $(H)^{3/2}$. Since the tailwater levels do not vary substantially at the higher heads, the plant sigma with the distributor set at Elev. 519.6 varies only with the inverse of H and sufficient submergence is assured.

k. The cavitation limits for the higher heads can be established for the selected setting by deriving a relationship or σ_c in terms of head, then entering the critical sigma curves on Figure S9 to estimate the corresponding value of HP_1 . This procedure is as follows:

$$\phi_{TH} = \frac{12.08}{(H_2)^{1/2}}$$

$$\sigma_c = \frac{H_b - H_v (\text{Distr. El.} - \text{T.W. El.} - a) - \text{safety}}{H_2}$$

By substituting known values and allowing a constant tailwater level at Elev. 540, this equation becomes:

$$\sigma_c = \frac{56 - 0.7 (H_2)^{1/2}}{H_2}$$

A summary of the maximum output limits is as follows:

H_2	ϕ_{TH}	σ_c	HP ₁	HP ₂
72	1.424	0.695	0.337	48,930
72	1.404	0.675	0.332	50,230

The limiting output of 50,190 HP can be developed at 74 feet and the higher heads without cavitation with distributor centerline Elev. 519.6.

1. The prototype dimensions of the principal parts and water passages of the turbines can be calculated from the dimensionless ratios shown in Table 4, Appendix C.